Abstract

Stability of Viscous Shock Profiles via Weighted Energy Estimates
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In this talk the use of weighted energy estimates in the proof of stablility of visous shock profiles is shown. A visous shock profile is a specific traveling wave solution

$$u(x,t) = \phi\,(x - st)$$

of the hyperbolic conservation law

$$\begin{aligned} &u_t + f(u)_x = u_{xx} \quad x \in \mathrm{IR},\ u \in \mathrm{IR}^n \\ &u(\pm\infty) = u_\pm \end{aligned} \tag{1}$$

(f' is assumed to have real eigenvalues, s and $u_\pm$ are subject to certain conditions). By stability we mean:

Theorem 1
Let $U_0 \in H^{2,2}$. If $|u_+ - u_-|$ and $||U_0||_{H^{2,2}}$ are sufficiently small, then the solution $u(x,t)$ to (1) with data $u(\cdot,0) = \phi + (U_0)_x$ exists for all times $t > 0$ and has

$$\lim_{t\to 0} \sup_x |u(x,t) - \phi(x - st)| = 0$$

In the course of the talk we will present the “integrated equation”, which is the starting point of the energy estimates. Then we will sketch a basic energy estimate for a scalar equation ($n = 1$) using a convexity assumption on f, show how this assumption can be removed by the use of weight functions and finally present the main ingredients for a similar treatment of a system ($n > 1$).