Introduction to the

Elastic Volume Weighted Moving Average (eVWMA)

Approximating the

Average Price Paid per Share

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Introduction

There have been many articles on suitable replacements or improvements of the widely used moving averages – e.g. by incorporating information about traded volume and time. In this article we will introduce the *elastic volume weighted moving average* which is a very natural replacement for standard moving averages since it can be seen as an approximation to *the average price paid per share* (if you are curious and want to see the definition, skip the motivation).

The averaging process we will introduce below differs from the one known:

- It does not refer to an underlying averaging period (e.g. 20 days, 38 days, 200 days) (which is always a hard to motivate and arbitrary choice). Rather it considers share volume to define the reference period.
- It incorporates information about volume (and possibly time) in a very natural way.
- Last not least: it can be derived from and seen as approximation to a statistical measure and thus has a solid mathematical justification.

Motivation

We will motivate the formula of the elastic volume weighted moving average (cf. [1]) by a step by step consideration of examples of widely used moving averages. For each of these we will point out the main defects. A rigorous mathematical derivation of the elastic volume weighted moving average can be found in [2].

Standard moving average

First consider a three-day moving average (we consider only three days to keep the example simple). In addition assume that there is only one trade each day (again, this is only to keep the example simple).

Example 1		
day/trade no.	price	volume
1	$10	100
2	$20	200
3	$30	300

Example 2		
day/trade no.	price	volume
1	$10	300
2	$20	200
3	$30	100

A standard three-day moving average is constructed by summing up the prices of the last three days and dividing by the number of days (here three). In both examples the three-day moving average is $20. The price of $10 at day one is balanced by the price of $30 at day three. However, volume suggests that in the first example the first day should be weighted less and the third day should be weighted more and vice versa in the second example.

Standard volume weighted moving average

This leads to the insight that the averaging should be volume weighted. A standard three-day volume weighted moving average is constructed by summing up the prices of the last three days, multiplied by the respective volume and divided by the total volume over the three days. The three-day volume weighted moving average in Example 1 is $23.33. The three-day volume weighted moving average in Example 2 is $16.67.

Example 3			
day/trade no.	price	volume	three-day volume weighted mav
1	$30	400	N/A
2	$40	400	N/A
3	$30	30	$35,68
4	$20	60	$36,90
5	$35	30	$26,92

The example presents the effect of the main defect of the three-day volume weighted moving average: information prior to the three days is not considered, even if volume suggests its importance. The three-day volume weighted moving average of the first three days is $35,68. Although on day four we see a lower price of $20, the three-day volume weighted moving average (now considering day 2, 3 and 4) rises. The reason is that we no longer consider day 1, and considering only day 2, 3 and 4 the high price at high volume on day 2 is becoming even more important as we “forget” day 1.

At day five the price rises to $35 (more or less the average value), but the three-day volume weighted moving average drops to $26,92. This is because on day 5 we forgot about day 1 and day 2 and consider only day 3, 4 and 5 among which day 4 is weighted most.

We forget about day 1 and 2, although volume suggests that they give the most reliable prices.

N-volume volume weighted moving average (defining the averaging period by volume)

This problem is solved by defining the averaging period by volume rather than days. We already hinted at this point since in our example we assume one trade a day and thus could also consider individual trades instead of days. Indeed, we have to calculate the average by considering trades since there are already many trades with different prices and different volumes each day[1]. So, a better averaging method would be to calculate the volume weighted moving average over the last trades with a total volume of N, where N is a fixed number that has an impact on the length of the averaging period like the number of days before. This N-volume volume weighted moving average should replace the K-day volume weighted moving average (where N is of the order of the volume of shares that are usually traded in a K-day period).

[1] Often this data is not available and all that is available is the closing price and the total volume for each day. In this case we should consider the closing price and the total volume as approximations by seeing them as a result of a single trade.

Elastic volume weighted moving average

We will now present another type of averaging that resembles the N-volume volume weighted moving average, but could be derived as approximation to the *average price paid per share*.

Motivation

We start by looking at the companies' IPO. Assume that 1000 shares have been issued at a price of \$10. So the shareholders paid a price of \$10 for a share. In the first trade 100 shares are traded at a price of \$15. This means that 900 shareholders hold shares for which they had paid \$10 and 100 shareholders hold shares for which they had paid \$15. The average price paid per share is thus

$$(900 \bullet \$10.00 + 100 \bullet \$15.00) / 1000 = \$10.50.$$

Example 4			
trade no.	price	Volume	eVWMA
IPO	\$10.00	1000	\$10
1	\$15.00	100	\$10.50
2	\$20.00	200	\$12.40
3	\$25.00	100	\$13.66

In the next trade 200 shares are sold for a price of \$20. But this information is not sufficient to know the new distribution of prices paid per share and thus calculate the average price paid per share. The reason is, that we do not know whether the 200 shares are sold by shareholders that originally paid \$10 for each share, or whether 100 shares are sold by shareholders that originally paid \$10 and the remaining 100 shares are sold by shareholders that originally paid \$15, or whether is was any other combination. Thus we have to make an approximating assumption about the average price the sellers paid for each of the 200 shares. A natural first approximation is that in average they paid the *previous average price paid per share* (which was \$10.5). The new (approximated) average price paid per share is

$$(800 \bullet \$10.50 + 200 \bullet \$20.00) / 1000 = \$12.40.$$

And after the third trade we find

$$(900 \bullet \$12.40 + 100 \bullet \$25.00) / 1000 = \$13.66.$$

Formulae

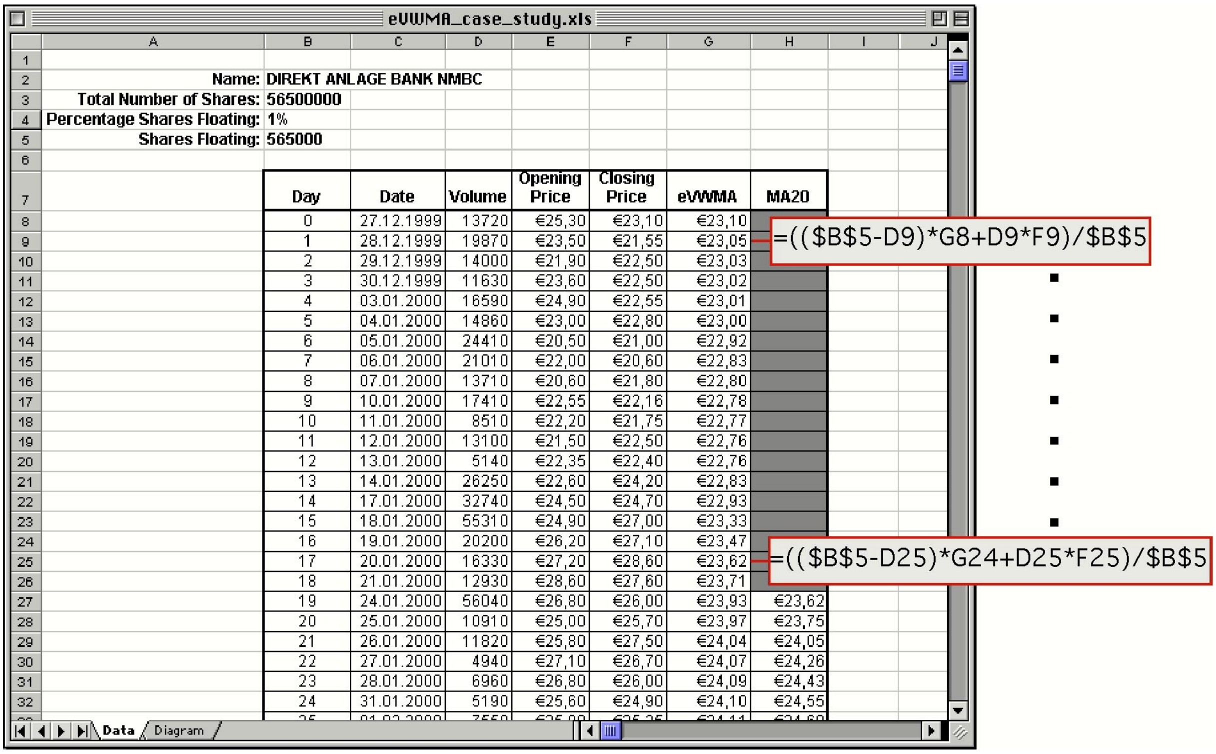
From the motivation we see that the *approximated average price paid per share* or *elastic volume weighted moving average* could be defined by a simple recursive formula that describes how to update the value after each trade: Let N be the number of shares floating, p be the price and v be the volume of shares traded in the next trade then we have

$$\text{new\_eVWMA} = [\ (N - v) \bullet \text{previous\_eVWMA} + v \bullet p\ ] / N\ .$$

Since very often a large amount of shares is locked by institutional investors, the number N should be chosen much smaller than the total number of shares. The choice of N influences the sensitivity of the averaging process and thus plays a similar role as the number of days in a K-days volume weighted moving average.

Implementation using spread sheets

Since *approximated average price paid per share* or *elastic volume weighted moving average* is defined by a simple "update rule" it can be easily implemented using spreadsheet calculations.



Name:	DIREKT ANLAGE BANK NMBC
Total Number of Shares:	56500000
Percentage Shares Floating:	1%
Shares Floating:	565000

Day	Date	Volume	Opening Price	Closing Price	eVWMA	MA20
0	27.12.1999	13720	€25,30	€23,10	€23,10	
1	28.12.1999	19870	€23,50	€21,55	€23,05	
2	29.12.1999	14000	€21,90	€22,50	€23,03	
3	30.12.1999	11630	€23,60	€22,50	€23,02	
4	03.01.2000	16590	€24,90	€22,55	€23,01	
5	04.01.2000	14860	€23,00	€22,80	€23,00	
6	05.01.2000	24410	€20,50	€21,00	€22,92	
7	06.01.2000	21010	€22,00	€20,60	€22,83	
8	07.01.2000	13710	€20,60	€21,80	€22,80	
9	10.01.2000	17410	€22,55	€22,16	€22,78	
10	11.01.2000	8510	€22,20	€21,75	€22,77	
11	12.01.2000	13100	€21,50	€22,50	€22,76	
12	13.01.2000	5140	€22,35	€22,40	€22,76	
13	14.01.2000	26250	€22,60	€24,20	€22,83	
14	17.01.2000	32740	€24,50	€24,70	€22,93	
15	18.01.2000	55310	€24,90	€27,00	€23,33	
16	19.01.2000	20200	€26,20	€27,10	€23,47	
17	20.01.2000	16330	€27,20	€28,60	€23,62	
18	21.01.2000	12930	€28,60	€27,60	€23,71	
19	24.01.2000	56040	€26,80	€26,00	€23,93	€23,62
20	25.01.2000	10910	€25,00	€25,70	€23,97	€23,75
21	26.01.2000	11820	€25,80	€27,50	€24,04	€24,05
22	27.01.2000	4940	€27,10	€26,70	€24,07	€24,26
23	28.01.2000	6960	€26,80	€26,00	€24,09	€24,43
24	31.01.2000	5190	€25,60	€24,90	€24,10	€24,55

Figure 1: Sample spread sheet

The sample spread sheet in Figure 1 contains date (column C), volume (column D), opening price (column E), and closing price (column F). Cell B5 contains the number of shares floating (here 1% of the total number of shares). Since we do not have information on volume and price of each single trade we use the daily volume and closing price as a proxy. In cell G8 we enter the initial value for the eVWMA. Starting from cell G9 we enter the update rule above.
A sample spreadsheet can be found at http://www.christian-fries.de/evwma .

A small case study

Using real world data, we will compare the elastic volume weighted moving average to a non-weighted moving average with fixed (20 days) time period.

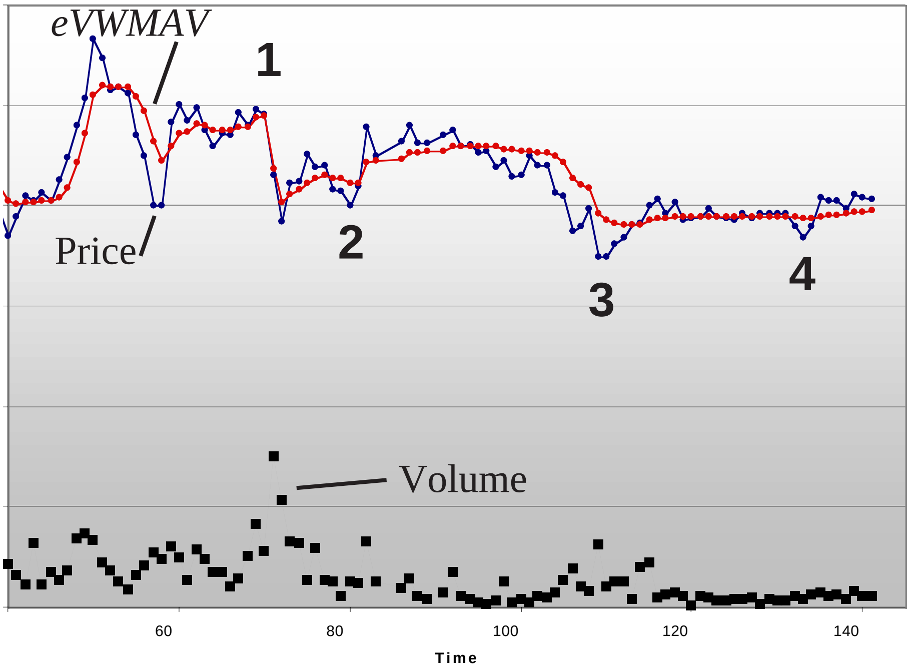


Figure 2: Stock price, eVWMA (sensitive) and Volume[2]

Figure 2 shows the stock price and a sensitive eVWMA. The eVWMA behaves like a moving average by smoothing out the price curve (3), on the other hand we see that at (1) the eVWMA changes abruptly and adjusts to the current price due to high volume (volume is plotted at the bottom of the diagram). The drop in price seen at (3) and (2) shows only small or very small impact on the eVWMA due to low volume. The drop in price at (4) is also not supported by volume and thus stays without effect on the eVWMA.

In Figure 3 we see differences in the behavior of eVWMA and an unweighted moving average: The sharp rise in price (1) is supported by huge volume. Thus the eVWMA adapts very quickly to this new situation, whereas the unweighted moving average (ignoring volume information) needs 12 to 20 days to adapt to the new situation. (Note: The sharp rise in price was due to new information becoming available at that time). Another difference is seen in (2): The undershooting price drop is not fully supported by volume and thus almost not visible in the eVWMA curve, whereas the (unweighted) moving average incorporates this information with a delay of 10 days and thus develops a bump.

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[2] Data of DRN.FSE (*Direktanlagebank*, 3rd biggest German online broker), 99 samples (February 21, 2000 to July 11, 2000).

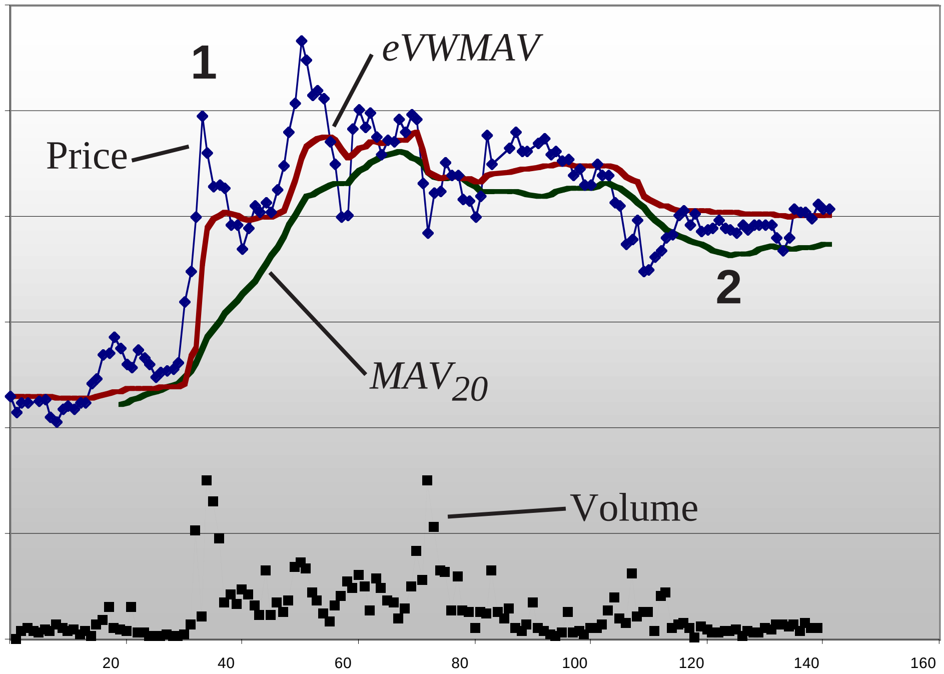

Figure 3: Stock price, eVWMA (medium sensitivity), 20 Days Moving Average (MAV_{20}) and Volume[3]

Outlook

There are many improvements possible using the approach presented. In our opinion the biggest advantage lies in the derivation from the *distribution of the average price paid per share.* The eVWMA is an approximation to the mean of this distribution. Starting from the *distribution of the average price paid per share* approximating formulas for several well-known statistical measures (e.g. standard deviation, moments, kurtosis, etc.) could be derived.
In addition it is possible to include the information about volume *and* time into the averaging process, cf. [2].

References

[1] Fries, Christian P.: Elastic Moving Averages, Technical Analysis of Stocks & Commodities, June 2001.

[2] `http://www.christian-fries.de/evwma`

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[3] Data of DRN.FSE (Direktanlagebank, 3rd biggest German online broker), 138 samples (December 12, 1999 to July 11, 2000).